Temperature effects on fatigue crack growth in polycarbonate

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The fatigue behaviour of a high strength thermoplastic, polycarbonate, has been investigated as a function of temperature. Fatigue crack growth properties were measured in the temperature range of 100 to 373 K and were analysed using a fracture mechanics approach. Fatigue behaviour was found to be related to the fracture toughness of the material. This correlation with fracture toughness was used to develop an empirical model based on the toughness for describing the effect of temperature on fatigue crack growth, and to consider fatigue in terms of the secondary losses of the polymer.

1. Introduction

The use of polymers as load-bearing materials necessitates an understanding of the behaviour of polymers under stress. Polymer mechanics has been studied for a broad variety of stress systems such as fatigue, the application of alternating stress below the yield or fracture stress. For engineering applications, the growth of fatigue cracks is particularly important.

The concepts of fracture mechanics have been applied to fatigue crack propagation in both metals and polymers [1-6], several models having been proposed to described fatigue behaviour. These macromechanical models are generally in the form proposed by Paris [1] in which the fatigue crack growth rate per cycle is related to an applied stress intensity or crack-tip displacement by a power law relationship.

Although there have been some attempts to model fatigue crack propagation behaviour from what is known of the continuum mechanics of the crack tip, there have been few integrated attempts to investigate this in terms of both fracture mechanics and dynamic properties. A few exceptions are the work of Williams [7] and Knauss [8] on viscoelastic fracture behaviour and the work of Marshall *et al.* [9] on glassy polymers. However, there has been no comprehensive correlation of fracture toughness, fatigue crack propagation and dynamic moduli in glassy polymers.

Furthermore, nearly all of the work on fatigue of polymers has been conducted at ambient conditions. Since polymers are used under a wide range of environmental states, this study was initiated in order to determine the effect of temperature on fatigue crack growth. In addition, the effects of two temperature dependent properties, the fracture toughness and the dynamic moduli, were investigated. The primary goals of the project were (1) to determine the validity of the fracture mechanics approach and the proposed models of fatigue crack propagation and (2) to investigate the micromechanism of crack growth in polymers.

2. Experimental

Extruded polycarbonate sheet (Lexan) was used in this study. Polycarbonate was chosen because of its high toughness, its amorphous structure, and its broad secondary relaxation peak. It was also chosen since impact data seemed to bear a relationship to both secondary losses and to microscopic fracture striations on the order of the crack-tip displacement in size [10].

An extensive characterization of polycarbonate was conducted. Fig. 1 shows the dynamic moduli as measured by a forced torsion pendulum at 1 Hz [11]. Differential scanning calorimetry indicated a glass transition temperature of 418 K.

The polycarbonate sheet was machined into two types of specimen geometries. Single edge

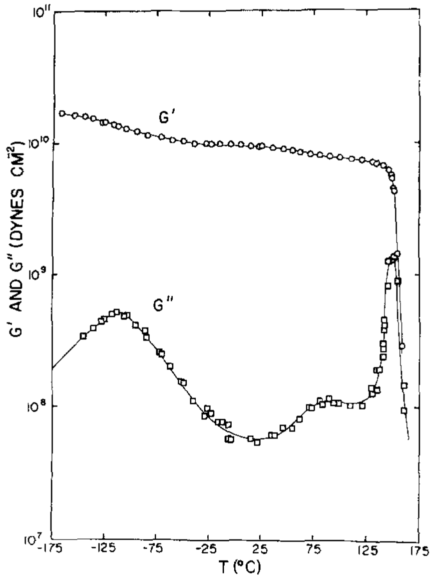


Figure 1 G' and G'' for polycarbonate at 1 Hz [11].

notch (SEN) specimens with dimensions 50.8 mm (2 in.) by 152.4 mm (6 in.) and 50.8 mm by 203.2 mm (8 in.) and grooved tapered cleavage (TC) specimens were tested. In all cases, the specimen thickness was 6.35 mm (0.25 in.). The stress intensity amplitude for these specimens is given by

$$\Delta K = \Delta\sigma\, a^{1/2} f(a/W) \tag{1}$$

where ΔK is the amplitude of the stress intensity, $\Delta\sigma$ is the applied stress amplitude, a is the crack length, W is the specimen width, and $f(a/W)$ is the geometric parameter. The geometric parameter for the single edge notch specimens was calculated using the collocation solution of Brown and Srawley [12]. For the grooved tapered cleavage specimens, a compliance method suggested by Marshall *et al.* [13] was utilized.

Fatigue and toughness tests were conducted using a closed loop hydraulic testing machine. This machine was equipped with a travelling microscope for measuring crack growth, a function generator and cycle counter, and an environmental chamber with a temperature range of 100 to 400 K. Tests below room temperature were conducted in a nitrogen atmosphere while tests at room temperature and above were conducted in air. All fatigue tests were run at 1 Hz in sinusoidal oscillation.

3. Theoretical background

The fracture mechanics approach to fatigue crack propagation, as originally used by Paris [1], gave a functional relationship between crack growth rate and change in stress intensity. In its simplest form, this is

$$\mathrm{d}a/\mathrm{d}N = \alpha(\Delta K)^m \tag{2}$$

where $\mathrm{d}a/\mathrm{d}N$ is the cyclic crack growth rate, ΔK is the amplitude of the stress intensity, and α and m are material and loading dependent constants. This model has been successfully applied to both metals and polymers [1-6].

In order to account for the effect of mean stress intensity K_m on fatigue, Arad *et al.* [6] proposed a similar model:

$$\mathrm{d}a/\mathrm{d}N = \alpha'\lambda^{m'}$$

$$\lambda = K_{max}^2 - K_{min}^2 = 2\Delta K K_m \tag{3}$$

where K_{max} and K_{min} are the maximum and the minimum cyclic stress intensities and α' and m' are constants. This model can be extended to include temperature effects, with respect to the change in elastic modulus, by considering the change in strain energy release rate, $\Delta\mathcal{G}$ [14]. The relation is

$$\mathrm{d}a/\mathrm{d}N = \alpha''(\Delta\mathcal{G})^{m''} \tag{4}$$

Since $\Delta\mathcal{G}$ may be written as

$$\Delta\mathcal{G} = \mathcal{G}_{max} - \mathcal{G}_{min}$$

and

$$\mathcal{G} = (1-\nu^2)K^2/E \tag{5}$$

for plain strain, where E is the tensile modulus, and ν is Poisson's ratio, the change in the strain energy release rate can be written as:

$$\Delta\mathcal{G} = (1-\nu^2)\lambda/E. \tag{6}$$

Using the elasticity relation

$$E = 2G'(1+\nu)$$

where G' is the shear modulus, the power law relation reduces to

$$\mathrm{d}a/\mathrm{d}N = \alpha''\left[\frac{(1-\nu^2)\lambda}{2G'(1+\nu)}\right]^{m''} \tag{7}$$

where α'' and m'' are constants. Thus, the temperature dependent modulus term can be normalized by plotting $\mathrm{d}a/\mathrm{d}N$ as a function of $\Delta\mathcal{G}$ from Equation 4 or as a function of λ/G' as indicated in Equation 7.

More recent analyses by Tomkins [15] and by Schwalbe [16] can be shown to reduce to a proportionality of the crack velocity to the crack tip displacement, with some modification for strain hardening. For example, Schwalbe's general result is

$$\mathrm{d}a/\mathrm{d}N = \frac{(1-\nu)(1-2\nu)^2}{\pi^2(1+n)\sigma_{ys}^2} \cdot \left[\frac{2\sigma_{ys}(1+\nu)(1+n)}{En^{n/1+n}}\right]^{1+n} \Delta K^2 \quad (8)$$

where n is the strain-hardening exponent. For ν equal to 0.37, which is the case for polycarbonate, this becomes

$$(\mathrm{d}a/\mathrm{d}N)_{n=1} = 0.065 \frac{\Delta K^2}{E^2} \quad (8a)$$

for the linear elastic result, and

$$(\mathrm{d}a/\mathrm{d}N)_{n=0} = 0.012 \frac{\Delta K^2}{\sigma_{ys}E} \quad (8b)$$

for the elastic–plastic result. This latter result is directly proportional to the crack-tip displacement.

It will be shown that such crack opening displacement and other continuum models [17] are inadequate for describing crack velocities in polycarbonate since they predict a value of 1 to 2 for the fatigue exponent m'' in Equation 7. However, this does not invalidate the fracture mechanics framework for describing crack velocities in terms of a prescribed set of material constants.

4. Results and discussion

Fatigue crack propagation data were obtained for polycarbonate over a temperature range of 100 to 373 K at 1 Hz. The log of the cyclic crack growth rate versus the log of the stress intensity amplitude at 25° C is plotted in Fig. 2. These results agree with the data of Arad *et al.* [5] and Hertzberg *et al.* [3]. In Fig. 3, the results at −50° C and 50° C are shown. The large differences both in the slope and in the position of the fatigue data are indicated on this graph.

As expected, the fracture mechanics approach is valid with respect to Equation 2. Variations in the data are probably due to physical irregularities at the crack tip since some difficulty was encountered in maintaining an even crack front. The non-linearity caused by water [2, 18] was particularly evident at 0° C. Other variations could arise from a change in the state of stress. For example, the conditions with 6.35 mm specimens at 100° C are more plane stress than plane strain.

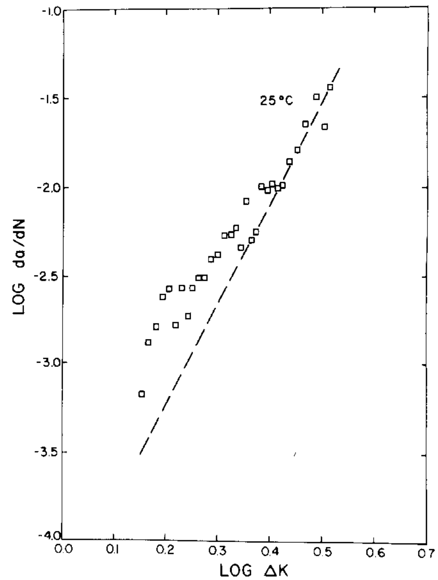


Figure 2 Fatigue crack growth of polycarbonate at 25° C, 1 Hz ($\mathrm{d}a/\mathrm{d}N$ (mm/cycle), ΔK(MPa m$^{1/2}$)). Dashed line is prediction curve using Equation 10.

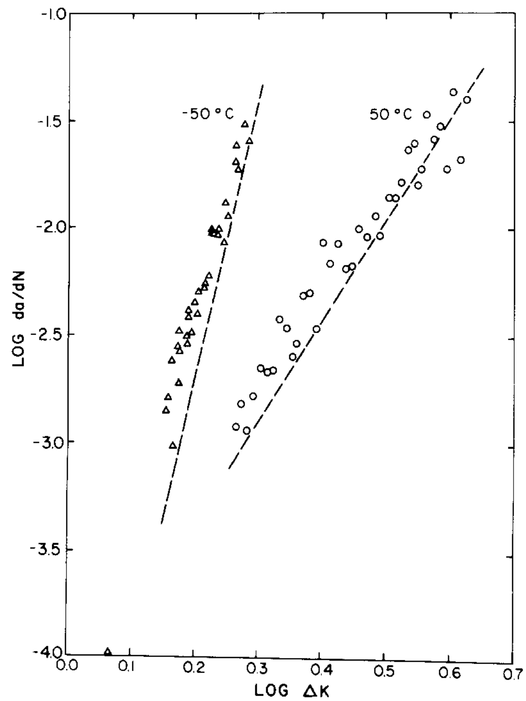


Figure 3 Fatigue crack growth at −50° C and at 50° C of polycarbonate at 1 Hz; Δ, −50° C, ○, +50° C. Dashed lines are prediction curves using Equation 10.

The difference in crack growth rate with temperature is shown in the least square fits at each temperature compiled in Figs. 4 to 6. These figures illustrate two important points. First, depending on the temperature, the stress intensity

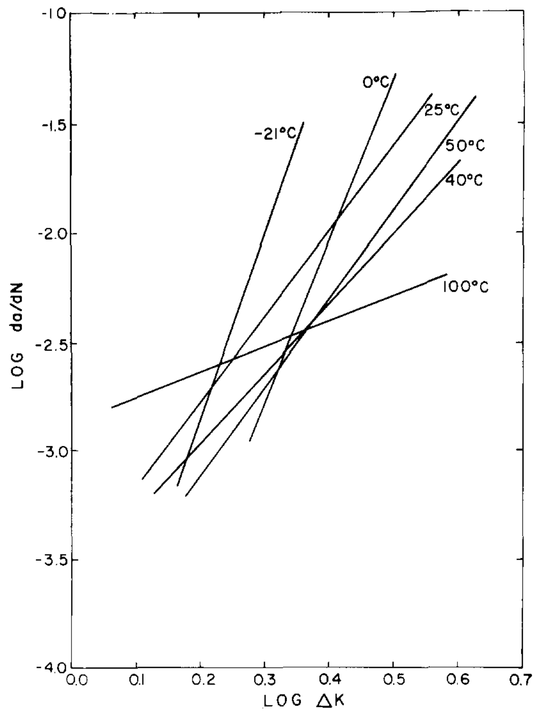


Figure 4 Fatigue crack growth of polycarbonate at 1 Hz. Temperature range: −21 to 100° C.

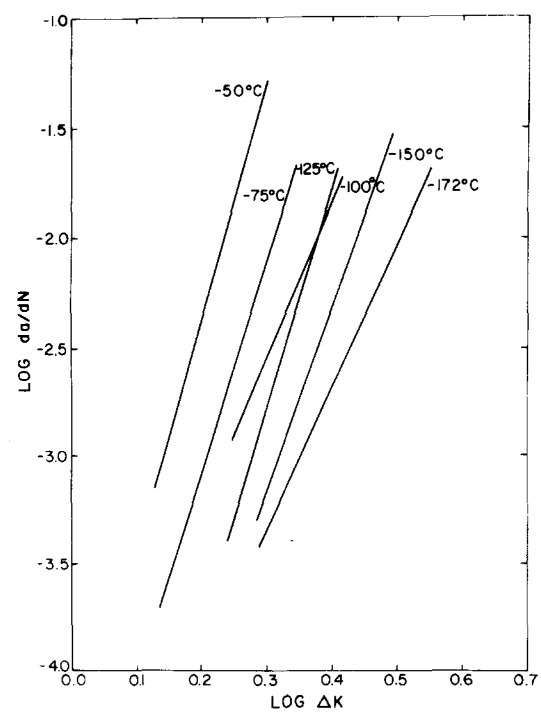


Figure 5 Fatigue crack growth of polycarbonate at 1 Hz. Temperature range: −50 to −172° C.

amplitude varies by a factor of as much as 2.3 at a constant crack growth rate. Fig. 6 also shows a shift in the direction of the crack propagation curves. Above −50° C, fatigue crack resistance

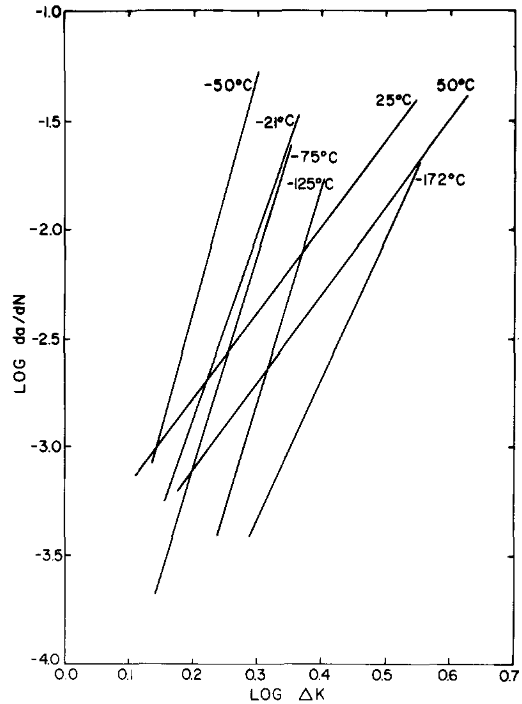


Figure 6 Fatigue crack growth of polycarbonate showing the effects of temperature on crack growth and the shift in fatigue behaviour at −50° C.

increases with increasing temperature. Below −50° C, fatigue crack resistance increases with decreasing temperature. These results agree with those of Kurobe and Wakashima [19] who investigated the fatigue crack growth of polycarbonate in the −50° C to 50° C temperature range.

Secondly, as shown in Figs. 4 and 5, the slopes of the fatigue curves change with temperature. Although some theoretical and most experimental results predict the value of the parameter m in Equation 2 to be from 2 to 5 [1, 2], m appears to be dependent on temperature ranging from a value of 10 at −50° C to 1 at 100° C and 6 at −172° C. Again, −50° C seems to be the transition point. Values of m and α are given in Table I.

TABLE I Least square parameters for $da/dN = \alpha(\Delta K)^m$

Temperature (°C)	α(mm/cycle)	m
−172	5.2×10^{-6}	6.5
−150	1.9×10^{-6}	8.4
−125	1.6×10^{-6}	10.0
−100	1.8×10^{-5}	7.2
−75	8.8×10^{-6}	9.8
−50	2.9×10^{-5}	10.7
−21	2.5×10^{-5}	8.6
0	9.4×10^{-6}	7.5
25	2.7×10^{-4}	3.9
40	2.4×10^{-4}	3.2
50	1.1×10^{-4}	4.1
100	1.4×10^{-3}	1.2

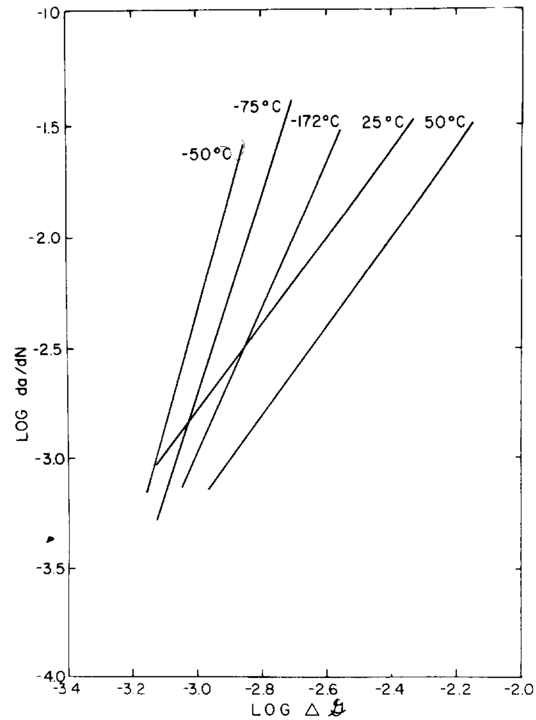
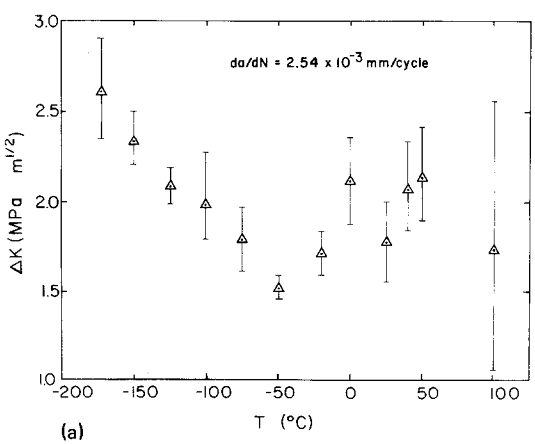


Figure 7 Fatigue crack growth of polycarbonate at 1 Hz using a strain energy release rate analysis (da/dN (mm/cycle), $\Delta\mathcal{G}$ (MPa m)).

In the theoretical background section it was suggested that one analysis which accounts for temperature effects is the strain energy release rate, $\Delta\mathcal{G}$. This correlation does give somewhat more normalized results as shown in Fig. 7. Since the effect of mean stress intensity was not specifically explored, the improvement of this model over the Paris model cannot be definitely established.



The transition in fatigue behaviour at $-50°$ C is displayed in Fig. 8a and b where the stress intensity amplitudes at two crack growth rates are plotted versus test temperature. These results were compared to the fracture toughness of polycarbonate. Using the data of Key *et al.* [20], Glover *et al.* [21], and this study, the fracture toughness, K_{IC}, is plotted as a function of temperature in Fig. 9. This graph also indicates a change in the fracture behaviour around $-50°$ C. Therefore, it appears that the mechanism which controls fracture instability in polycarbonate also controls its fatigue behaviour. This result is further illustrated in Fig. 10 which is a plot of the stress intensity amplitudes at constant crack growth rate, $(\Delta K)_{da/dN}$, versus the fracture toughness of the polymer at the same temperature.

This correlation between fatigue crack growth and fracture toughness can be used to predict fatigue crack propagation curves. Assuming a linear relationship, least squares lines were determined as shown in Fig. 10. Using this linear relation between $(\Delta K)_{da/dN}$ and K_{IC} with Equation 3 leads to

$$da/dN_1 = \alpha(K_{IC}b_1 + c_1)^m \qquad (9)$$

where da/dN_1 is a particular crack growth rate, and b_1 and c_1 are the slope and the intercept of the linear relation between ΔK at da/dN_1 and K_{IC}.

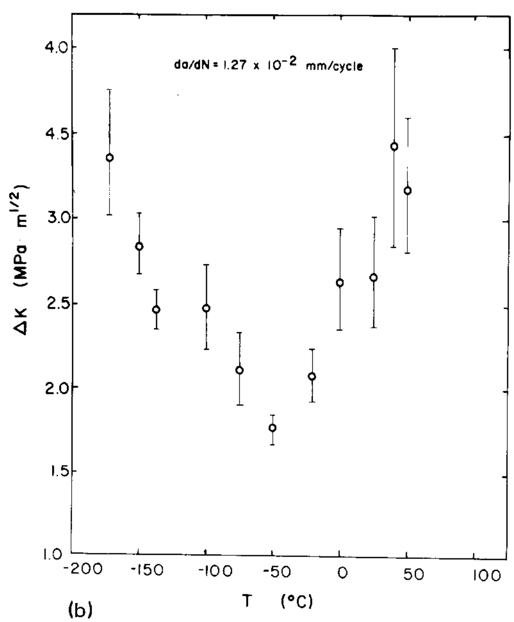


Figure 8(a) stress intensity amplitude with 95% prediction limits as a function of temperature at $da/dN = 2.54 \times 10^{-3}$ mm/cycle. (b) Stress intensity amplitude with 95% prediction limits as a function of temperature at $da/dN = 1.27 \times 10^{-2}$ mm/cycle.

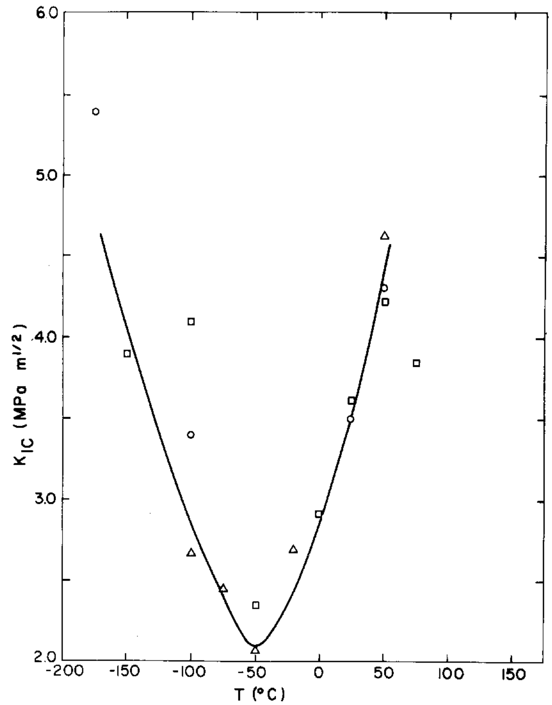


Figure 9 The fracture toughness of polycarbonate as a function of temperature □ [120]; ○, [21]; △, present results).

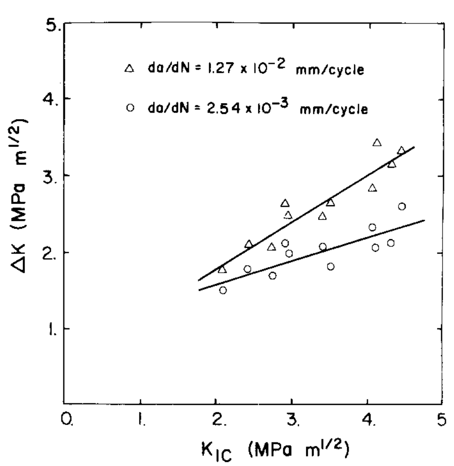


Figure 10 Stress intensity amplitude as a function of fracture toughness at two crack growth rates.

Extending this relation to other crack growth rates results in parameters b_2, c_2. for da/dN_2, etc. Thus, the values of α and m can be determined as a function of the fracture toughness or the test temperature using Equation 10 and Fig. 9. The experimental and the predicted values of the parameter m are shown in Fig. 11. Fatigue crack propagation curves can then be constructed

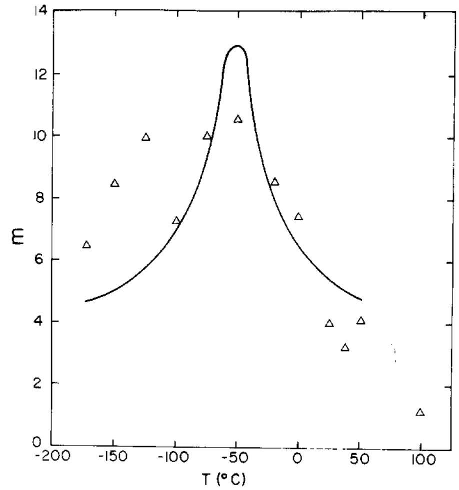


Figure 11 Experimental and predicted values of the crack growth exponent m as a function of temperature.

from these values by using a relation similar to Equation 2:

$$da/dN = a_p \Delta K^{m_p} \tag{10}$$

where α_p and m_p are the predicted values of α and m. The predicted crack growth rates are within a factor of 4 and generally within a factor of 2 for all the temperatures studied. Predicted curves at 25° C, −50° C, and 50° C are shown on Figs. 2 and 3.

Although the effects of frequency and mean stress are not included in this empirical approach, this model is more satisfactory than theoretical models such as Equation 8 which do not represent either the magnitude or the trend in the data. The significance of the model and Figs. 8a and b, 9, 10 and 11 is that fatigue crack propagation data are uniquely related to fracture toughness data, a situation which does not arise in other material systems [2].

The other interesting aspect of these data is the possible reason for this correlation. If the energy dissipation mechanism arising from the large secondary losses controls both the fatigue and the fracture process when tested at similar strain-rates, then one might expect a correlation. It was previously suggested [21] and later shown [22] that fracture toughness correlated with the integrated losses, at least at temperatures well below T_g. It was assumed that the adiabatic temperature rise at the rapidly moving crack front could be as much as 100 K, and, therefore, an integrated loss, L, can be defined as

$$L = \int_{T}^{T+100\mathrm{K}} G'' \mathrm{d}T. \qquad (11)$$

Averaging L over the 100 K rise, from data in Fig. 1, and determining the work per unit fracture area over a unit fracture region (e.g. the crack-tip displacement), lead to an excellent correlation between the observed integrated loss and the fracture toughness data. One might expect a similar correspondence between the integrated losses and the fatigue crack propagation data.

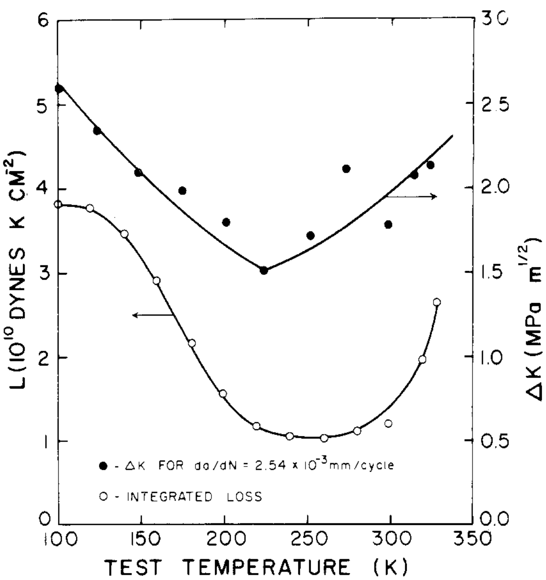


Figure 12 Comparison of dynamic losses and fatigue crack propagation as a function of test temperature.

Picking the ΔK curve for $\mathrm{d}a/\mathrm{d}N = 2.5 \times 10^{-3}$ mm/cycle, which gives a calculated crack-tip displacement of about 2.5×10^{-3} mm, one can show the correlation in Fig. 12. Due to the adiabatic temperature rise, there could be a small shift in the ΔK curve to the right which would make the integrated loss minimum very close to the minimum in the fatigue data. Considering the absolute magnitudes, there is a factor of 3.8 decrease in the integrated loss curve while there is a factor of 1.7 decrease in fracture toughness. Since energy release rate is proportional to K^2, this represents nearly a factor of three in energy. On the other side of the minimum, a factor of 2.6 rise in the integrated loss corresponds to a factor of 2.4 rise in $(\Delta K)^2$. Because of this correlation between integrated losses, fracture toughness and crack propagation data, it appears that secondary losses control both fracture toughness and fatigue crack propagation in this glassy polymer at low temperatures.

5. Conclusions

(1) The fracture mechanics approach is valid for the fatigue and the fracture behaviour of polycarbonate over the temperature range 100 to 330K.

(2) Plots of log $\mathrm{d}a/\mathrm{d}N$ (crack velocity) versus log ΔK (stress intensity amplitude) or log $\Delta \mathcal{G}$ (strain energy release rate amplitude) result in relationships of the form,

$$\mathrm{d}a/\mathrm{d}N = \alpha(\Delta K)^m; \mathrm{d}a/\mathrm{d}N = \alpha''(\Delta \mathcal{G})^{m''}$$

either one of which describes crack velocities in the range of 2.5×10^{-4} to 2.5×10^{-2} mm/cycle.

(3) A minimum in both $(\Delta K)_{\mathrm{d}a/\mathrm{d}N}$ for constant fatigue crack velocity and K_{IC} is found at 223 K which means that the material is least tough at this temperature.

(4) There is a linear relationship between $(\Delta K)_{\mathrm{d}a/\mathrm{d}N}$ and K_{IC} over the temperature range of 100 to 330 K.

(5) The interrelationship between ΔK, K_{IC} and m allows crack propagation velocities to be predicted from facture toughness data.

(6) The unique relationship between K_{IC} and fatigue crack propagation data appears to arise from a dependency on the integrated losses which control low temperature toughness.

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